Mapping static pressure gradients in a tornado-like vortex

Fred L. Haan, Jr. [1]

[1]*Calvin University, Grand Rapids, USA, frederick.haan@calvin.edu*

SUMMARY:
The pressure gradient field of a tornado vortex has a complex 3D structure and warrants in-depth study. This study developed tools to map the gradient fields of a vortex along with their constituent velocity gradients. While much attention has been paid to pressure gradients in the vertical direction in the past, this work shows that all three directions generate non-negligible forces. A non-dimensional pressure gradient force parameter was developed to assess whether the static pressure is capable of moving an object – in this case a vehicle. Not only was the static pressure gradient field found to vary radially and vertically, the influence of the translation of the vortex was found to be non-negligible. This suggests that static pressure fields should include the unsteady effects of translation. The tool developed for this study can be used to assess pressure gradient fields in any vortex velocity field whether it was generated analytically or experimentally.

Keywords: tornado, static pressure, pressure gradient

1. INTRODUCTORY COMMENTS ON STATIC PRESSURE FIELDS

Tornado effects on structures have been studied extensively in recent years using laboratory tornado simulators, computational simulations, and field damage surveys. It has proved difficult, however, to distinguish loading effects caused by the velocity field interacting with the structure from loading effects caused by the static pressure field that is generated/dependent on the velocity field. The gradients in the static pressure field can vary in importance for building aerodynamics depending on the relative scale of the tornado to the building. However, for initiation of flight of debris, these pressure gradients play a vital role. The motivating work for this study was how vehicles are moved by tornado winds. A tool was developed for determining the relative importance of pressure gradients in all three directions in addition to determining the underlying velocity-field dependences of each pressure gradient.

2. ANALYTICAL FRAMEWORK FOR PRESSURE GRADIENT MAPPING

To map the pressure gradient for a tornado vortex, this study started with the analytical velocity model of Baker et al. (2020). This model provides expressions for circumferential, radial, and vertical velocities in three-dimensional polar coordinates. The model is axisymmetric, so it has not circumferential gradients (θ-direction). This section describes how this velocity field was used to calculate pressure gradients and how those pressure gradients were normalized to help provide a sense of the relative importance of the gradients in each direction and of the constituent parts of each gradient.

2.1. Pressure gradient calculation

The inviscid Euler equations were used for estimate the pressure gradients in each direction, r, θ, and z. Equations (1) through (3) below show the equations arranged to solve for the pressure gradients but also showing that the velocity gradients with respect to θ are ignored since the tornado model is axisymmetric. In these equations, u, v, and w are the circumferential, radial, and vertical velocity components, respectively.

$$\frac{\partial p}{\partial r} = -\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \cancel{\frac{v}{r}\frac{\partial u}{\partial \theta}} - \frac{v^2}{r} + w\frac{\partial u}{\partial z}\right) \tag{1}$$

$$\frac{\partial p}{\partial \theta} = -\rho r\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial r} + \cancel{\frac{v}{r}\frac{\partial v}{\partial \theta}} + \frac{uv}{r} + w\frac{\partial v}{\partial z}\right) \tag{2}$$

$$\frac{\partial p}{\partial z} = -\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r} + \cancel{\frac{v}{r}\frac{\partial w}{\partial \theta}} + w\frac{\partial w}{\partial z}\right) \tag{3}$$

The pressure derivatives in the above equations were estimated in an rz-plane that passed through the center of the vortex. Second order finite difference expressions were used to calculate the derivatives on the right hand sides of equations (1) through (3) from the Baker et al. (2020) velocity expressions. To account for vortex translation, a constant linear velocity was added to these velocity expressions.

2.2. Normalization scheme for pressure gradients

To estimate the physical significance of the pressure gradients, a particular normalization scheme was developed. Because one application of this work will be for debris flight initiation – particularly vehicle motion initiation – the scheme involves vehicle parameters. Fig. 1 shows a representation of a vehicle as a rectangular solid. The vertical projected area is given as A and the pressures on the upper and lower surfaces are given as p_2 and p_1, respectively.

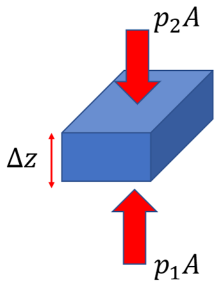

Figure 1. Example of figure.

The normalization scheme comes from normalizing the pressure force by the weight of the vehicle. The force, F, due to the pressure would be:

$$F = (p_2 - p_1)A = \Delta pA = \frac{\partial p}{\partial z}\Delta zA \tag{4}$$

Normalizing F by the weight of the vehicle, W, defines a non-dimensional pressure gradient force given the subscript z here since it is derived from the gradient in the z direction:

$$P_z = \frac{F}{W} = \frac{\frac{\partial p}{\partial z}\Delta z A}{W} \tag{5}$$

Similar gradient parameters can be formed for the radial and circumferential directions. They are shown in Eqn (6) below. A small vehicle was assumed for this study with a weight of 9400 N and with height, width, and length of 1.2m, 1.7m, and 4m, respectively.

$$P_r = \frac{\frac{\partial p}{\partial r}\Delta r A}{W} \qquad P_\theta = \frac{\frac{\partial p}{\partial \theta}\Delta \theta A}{W} \tag{6}$$

4. PRESSURE GRADIENT MAPS

Fig. 1 shows the pressure gradient force coefficients for a single-celled vortex with maximum circumferential wind speed of 75 m/s and a translation speed of 25 m/s. The vertical pressure gradient force coefficient is the largest with magnitudes nearly 40% of that needed to lift a vehicle (see section 3 for description of normalization). It should be noted that the radial and tangential gradient forces are not negligible but are nearly half of the magnitude of the vertical forces.

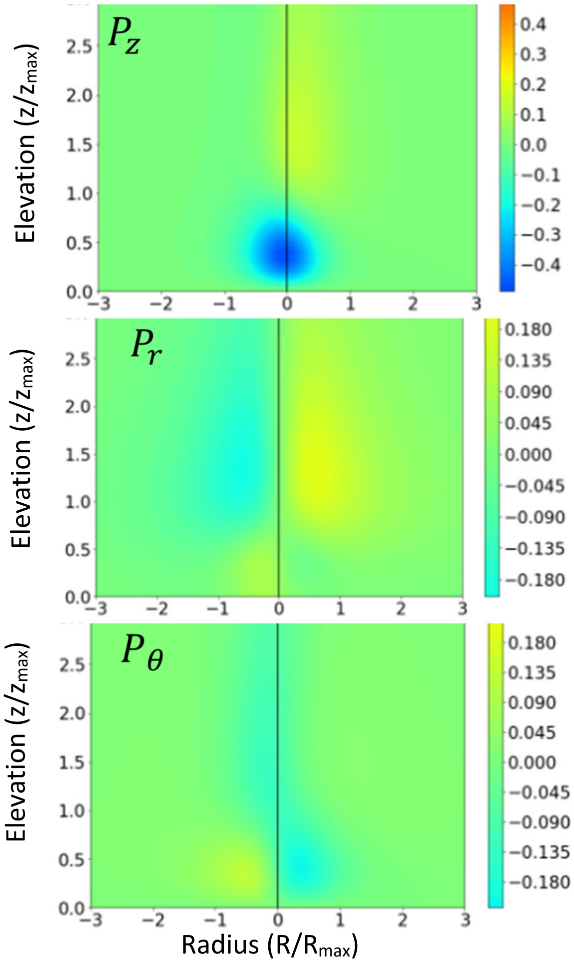

Figure 2. Pressure gradient force coefficients in each direction for a single-cell vortex with max circumferential velocity 75 m/s, translation velocity of 25 m/s, radius of max winds (R_{max}) of 30m and elevation of max winds (z_{max}) of 10m. A coefficient of 1.0 would be a large enough force to move a vehicle.

To better understand the underlying flow physics of these pressure gradients, they were also broken down into their component velocity gradients. Examples of these component plots are shown in Fig. 3 for the vertical pressure gradient force coefficient, P_z. The largest contribution to the vertical force comes from the vertical velocity gradient, but it should be noted that the contribution from the time derivative is nearly 12% of that of the vertical velocity gradient.

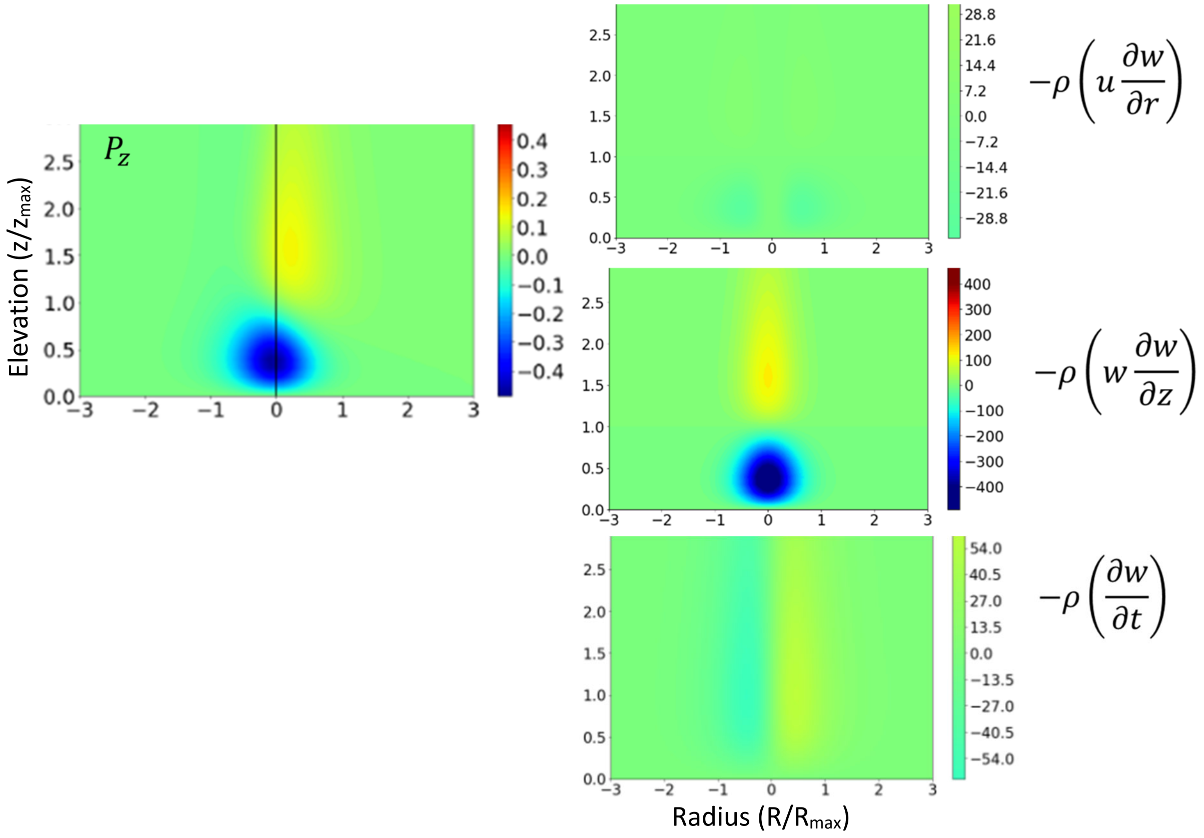

Figure 3. Plots of the velocity gradient components of the P_z coefficient. While P_z is dimensionless, the component plots have units of pressure gradient [Pa/m] to show relative magnitudes.

7. CONCLUSIONS:

The pressure gradients in a tornado have a complex 3D nature and deserve in-depth study. The current work suggests also that the influence of the vortex translation velocity (and the accompanying unsteady velocities) on the pressure gradients should not be ignored.

ACKNOWLEDGEMENTS

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